Errata for Vector and Geometric Calculus Printings 1-4

Note: “p. m (n)” refers to page m of Printing 4 and page n of Printings 1-3.

p. 56 (54), an omission, not an error. New second paragraph after Theorem 4.6: “The tangent space is a vector space (LAGA Exercise 8.12).”

p. 58 (55), caption of Figure 4.9. “f’ maps” → “f’ maps”.

p. 62 (58), Definition 5.2. Add a footnote after the first line: “i.e., the scalar coefficients of F are differentiable.”

p. 75 (69), 2nd paragraph. “Eq. (6.26)” → “LAGA Eq. (6.26)”.

p. 79 (73), Problem 5.4.1a. “Eq. (6.25)” → “Eq. (5.19)”.

p. 166, Theorem 11.8. “Let x(u(t), v(t))” → “Let x(u_1(t), u_2(t)).”

Errata for Vector and Geometric Calculus Printings 1-3

p. 30, Theorem 3.10, first line of proof:
\[(\partial_i f_1(x), \ldots, \partial_i f_n(x)) \rightarrow (\partial_i f_1(x), \ldots, \partial_i f_m(x)).\]

p. 40, just before the problems. “series of a vector valued function f centered” → “series of an f centered”.

p. 44, first line. “differentiable” → “continuously differentiable”

p. 54, Theorem 4.5 statement. “on C” → “on U”

p. 55, change bottom to

Let f be a 1-1 map between manifolds of equal dimension. Then the tangent map f’ maps T_p to T_{f(p)}.

p. 58, Theorem 4.7 statement. “on S” → “on U”

p. 64, Problem 5.2.11. \(\nabla \wedge e = -\partial_t \rightarrow B\nabla \wedge e = \partial_t B.\)

p. 76, Problem 5.5.2. New Part (b): Define a directional derivative for fields defined on a surface by \(\partial_h f(p) = (h \cdot \partial)f(p)\) (Definition 11.14). Compute \(\partial_t t\) on the equator. Ans. \(-\sin \theta i + \cos \theta j)/\rho.\)

Note that at the equator t is in the tangent plane but \(\partial_t t\) is not.

p. 82, Theorem 6.7 statement. \(g(x_0) = c \rightarrow g(x) = c.\)

p. 105, Problem 7.4.3. Field should be \(e^z (\sin(xy) + y \cos(xy))i + xe^z \cos(xy)j\)

p. 116, Exercise 8.8. The answer is \(\pi(e^4 - 1).\)

p. 132, Corollary 10.3. “Let f be a multivector field” → “Let f be a vector field”

p. 134, Problem 10.2.6c. \(\oint_C e \cdot ds = \partial_t \int_S B \cdot dS \rightarrow \oint_C e \cdot ds = -\partial_t \int_S B \cdot dS\)

p. 186. Add “65” and “66” to gradient entry. Add “98” to curl entry. Add “124” to divergence entry.
Errata for *Vector and Geometric Calculus*
Printings 1-2

p. 14, line 8: Delete "in".
p. 23, line 4: Change the period after "tangents" into a comma.
p. 28, Exercise 3.9: Change "Eq. (3.23)" to "Eq. (3.18)".
p. 30, Exercise 3.13. \( f: U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m \rightarrow f: \mathbb{R}^n \rightarrow \mathbb{R}^m \).
p. 38, Problem 3.4.12c. Change to "The ideal gas law is \( pv = nRT \), where \( n,p,v,T \) are the number of moles, pressure . . . .".
p. 43, last two lines. \( f((a,b)) = 0 \rightarrow f((a,b)) = (2,2) \).
p. 49, Definition 4.1. "Let \( q \in A \) and set \( p = x(q) \)." → "Let \( t \in A \) and set \( p = x(t) \)."
p. 55, Figure 4.9, caption. "onto \( S \) → "to \( S \).
p. 59, Corollary 5.7. \( f'\ast(b) \rightarrow f'\ast x(b) \).
p. 59, line -5: Change "Eq. (5.2) and Eq. (3.6)" to "Eq. (5.2) and Eq. (3.23)".
p. 64, bottom. Problem 4.3.12 → LAGA Problem 4.3.12
p. 69, Eq. (5.17). \( \partial x_i \partial w_j e_i \rightarrow \partial x_\ell \partial w_j e_\ell \)."
p. 72, Exercise 5.23. Equations should read
\[ \hat{\phi} = \cos \phi (\cos \theta i + \sin \theta j) - \sin \phi k, \quad \hat{\theta} = -\sin \theta i + \cos \theta j. \]
p. 75, footnote. \( \mathbb{R}^3 \rightarrow \mathbb{R}^n \).
p. 76, Problem 5.5.1b, second printing only. "even though \( \hat{t} \) is" → "even though \( \hat{t} \) is".
p. 79, Theorem 6.5. A much better proof:
Proof. Since \( \nabla f(x) = 0 \), \( \partial_i f(x) h_i = 0 \). And \( \partial_{ij} f(x) h_i h_j > 0 \) for \( h \neq 0 \), since \( H f(x) \) is positive definite. Then \( \partial_{ij} (f(x + t^* h) h_i h_j > 0 \) for small \( t^* h \neq 0 \), since the partial derivatives are continuous at \( x \). The theorem now follows from Eq. 3.2. \( \square \)
p. 81, Problem 6.1.2c. \( \lim_{(x,y) \rightarrow \infty} \rightarrow \lim_{(x,y) \rightarrow \infty} f(x, y) \).
p. 90, equation mid-page, \( \int_{[a,b]} a f dx = a \int_{[a,b]} f dx \rightarrow \int_{[a,b]} c f dx = c \int_{[a,b]} f dx. \)
p. 91, after the sentence beginning with Think of. "Divide \( C \) into infinitesimal parts. Multiply the value of \( F \) on each part by the infinitesimal length \( ds \) of the part. Add to form the integral."
p. 96, note at the bottom of the page. "here here" → "here".
p. 98, Problem 7.3.4c. Misplaced "": "\( F(x(u,v) x_u(u,v)) \)" → "\( F(x(u,v)) x_u(u,v) \)".
p. 100, Exercise 7.21. "Theorem 7.12" → "Theorem 7.10".
p. 100, proof of Theorem 7.13: ‘it independent’ → “it is independent”.
p. 101, Exercise 7.24b. “not conservative” → “not conservative in \( \mathbb{R}^2 - \{0\} \).

p. 102, following Definition 7.16. “All simple closed curves” → “All closed curves”.

p. 103, line -7. Switch “m” and “M”.

p. 103, line -5: Change “Eq. (7.10)” to “(Eq. (7.14))’.

p. 107, line -3: Change “set open” to “open set”.

p. 112, Exercise 8.2. \( \int_{v=0}^{1} \rightarrow \int_{v=0}^{2} \).

p. 124, Problem 9.2.3: “scalar + trivector” → “vector + trivector”.

p. 131, Fig. 10.6: Arrows should be reversed, as the \( M_i \) are “oriented clockwise”.

p. 136, second line of the proof of Corollary 10.5: \( (-1)^{2\times2} \rightarrow (-1)^{2\times1} \).

p. 142, below Corollary 10.10. \( f(x) = \int_{a}^{\bar{x}} f'(t)dt + f(a) \).

p. 146, lines 12 and 17: Change “Theorem 4.3b” and “Theorem 4.3” (both) to “Eq. (4.6)”.

p. 148, Theorem 11.5, Proof. \( \ddot{x} \rightarrow \dot{x} \), twice

p. 151, line above Def 11.10: Change “Definition 5.23” to “Definition 5.15”.

p. 158, middle displayed line in the proof of Theorem 11.25. Drop the middle term.


p. 162, line 5. Delete “is”.
Errata for *Vector and Geometric Calculus*

Printing 1

Due to publisher error the shading in several of figures is washed out in some copies of the book. The correct shading is shown below. I think that if Figure 2.1 is OK in your book, then all figures are.

p. 4, Exercise 1.1. “$c_{ij} \rightarrow c_{ik}$”.

p. 6, Problem 1.1.1. “$x(t) \rightarrow x(\theta)$”.
p. 8, Exercise 1.11. Delete “We will do this often.” Add “This will allow us to specialize formulas for surfaces defined parametrically to surfaces defined by $z = f(x, y)$. Exercise 5.25 is an example.”

p. 11, Exercise 1.14. “$x \neq 0$” → “$x > 0$”.


$\phi = \arccos(z/r) \rightarrow \phi = \arctan(r/z)$.

p. 22, bottom. Remove “The definition shows that $\partial_{i}F$ has the same grades as $F$.” Parts disappear if their partial derivative is zero.

p. 23, Exercise 3.1. “$REm$ to $\mathbb{R}^n”$ → “$Rm$ to $\mathbb{R}^n”.

p. 33, Problem 3.2.1. Append the sentence “Then for fixed $x$, the differential is the linear transformation $h \mapsto f'(x)h$.”

p. 33, Problem 3.2.3. $(\rho, \theta, \phi) \rightarrow (\rho, \phi, \theta)$

p. 34, first and second displayed equations should read

$x, h \in \mathbb{R}^n \Rightarrow (g \circ f)(x) \in \mathbb{R}^p \Rightarrow (g \circ f)'(h) \in \mathbb{R}^p$,

$x, h \in \mathbb{R}^n \Rightarrow f'(x)(h) \in \mathbb{R}^m \Rightarrow (g_{f(x)} \circ f'(x))(h) \in \mathbb{R}^p$.

p. 34. Line should read $4 = \ldots + [g'(R(h)) \mid h] + S(kh)\|kh\|].$

p. 34. Replace the end of the page with the following:
The added phrase “divided by $|h|$” is the reason for the changes.
To finish, we show that the term in brackets above, divided by $|h|$, approaches zero with $|h|$. First, using the continuity of $g'$ (Theorem 2.10),

$$\lim_{h \to 0} g'(R(h)) = g'(\lim_{h \to 0} R(h)) = g'(0) = 0.$$ Second, with $|f'|_\sigma$ the operator norm of $f'$,

$$|kh| \leq |f'(h)| + |R(h)||h| \leq |f'|_\sigma|h| + |R(h)||h|.$$ Thus, since $(h \to 0) \Rightarrow (kh \to 0) \Rightarrow (S(kh) \to 0)$, $\lim_{h \to 0} |S(kh)||kh|/|h| = 0$.

p. 37, statement of Theorem 3.16. “Then the inverse function $(f')^{-1}$” → “Then the inverse function $f^{-1}$”.

p. 40, following Theorem 3.13: “In other words, $\partial_{h}f$ is linear in both $h$ and $f$.”

p. 40, Problem 3.3.5. The variable names I used lead to confusion. Change to $f(x, y) = (x \cos y, x \sin y)$. And add “(All coordinates are cartesian.)”

p. 40, Problem 3.3.3. “continuity of $f$ at $x$.” → “continuity of $f$ at $x$.”

p. 41, Statement of Theorem 3.19. “has a differentiable inverse” → “has a continuously differentiable inverse”.

p. 42, third line. Remove “there is a neighborhood of each $y_i$ in which”.

p. 45, Exercise 3.6.1b. “Determine $\partial \phi/\partial x$.”

p. 48, below Eq. (4.3). “of higher dimension” → “in higher dimensions”.
pp. 49-55. Equation (4.4) in Section 4.1 established the notation of m-dimensional manifolds $M$ as subsets of $\mathbb{R}^n$. However, Sections 4.2 and 4.3, while mostly internally consistent, are inconsistent with this notation. The following changes remove the inconsistency:

- p. 49, 2nd paragraph: "a curve $C$ in $\mathbb{R}^n".
- p. 50, Theorem 4.2: $m$ to $n$.
- p. 51, Theorem 4.4: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 52, Theorem 4.5: $\mathbb{R}^m$, $\mathbb{R}^n$ to $\mathbb{R}^n$.
- p. 53, First sentence and left part of Figure 4.6: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 53, Definition 4.6: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 53, line following Eq. (4.11): $m$ to $n$.
- p. 55, Theorem 4.7: $\mathbb{R}^m$ to $\mathbb{R}^n$, $\mathbb{R}^n$ to $\mathbb{R}^n$.

p. 49, Paragraph 2. "Let $x(t)$ parameterize a curve $C$. Then $x$ has a nonzero differential (p. 49). By Problem 3.2.1 it is . . ."

p. 50, line after Theorem 4.3. "Definition 4.5" → "Eq. (4.5)"

p. 52, Theorem 4.5. "$f'_p$ is one-to-one." → "$f'_p$ restricted to $T_p$ is one-to-one."

p. 52, Problem 4.2.4.
  a. Show that components of $\Omega$ must have grade 2 or 3.
  b. Show that components of $\Omega$ must have grade 0 or 2.

p. 53, Eq. (4.10). "$\lim_{h \to 0}$" → "$\lim_{h \to 0}$"

p. 53, sentence below Definition 4.6. "Recall that the differential $x'_q$ is one-to-one and maps linearly independent vectors . . . ."

p. 55, Theorem 4.7. "$f'_p$ is one-to-one." → "$f'_p$ restricted to $T_p$ is one-to-one."

p. 56, Problem 4.3.1b. $x_u \wedge x_v \to x_\phi \wedge x_\theta$.

p. 57, Definition 5.1. "Let $M$ be a manifold in $\mathbb{R}$. A field on $M$ is a function defined on $M$ whose values are in $G^n$."

p. 62, Problem 5.2.5. Change to $\nabla \cdot (xf(|x|)) = nf(|x|) + |x|f'(|x|)$.

p. 64, Problem 5.2.11b. Remove the word "both".

p. 67, Exercise 5.14a. $\nabla \cdot f = \partial_1 f_1 + \partial_2 f_2$.

p. 71, Figure 5.7. "$x(c_1, c_2, c_3)$" → "$x(c_1, c_2, u_3)$".

p. 71, Paragraph 4. In general: (i). Each basis vector $x^k$ is orthogonal to the surface formed by fixing the coordinate $u_k$. (ii). Each basis vector $x_j$ is tangent to the curve which is the intersection of the two surfaces formed by fixing in turn the coordinates other than $u_j$.

p. 72, formula for $\nabla f$ in cylindrical coordinates. "$r^{-1} \partial f_\theta$" → "$r^{-1} \partial_\theta f$".

p. 74, Problem 5.4.2. Better: Hint: If $B$ is a blade, then $B^{-1} = B/B^2$, where $B^2$ is a scalar.

p. 74, Problem 5.4.5. Delete. Renumber Problems 5.4.6-5.4.8 to 5.4.5-5.4.7.

p. 75, line 3. "manifolds in $\mathbb{R}^m$ → "manifolds in $\mathbb{R}^n$".

p. 76, Problem 5.5.1. "to the unit sphere" → "in $\mathbb{R}^2$"
"$D = P_T(\partial)$" → "$DF = P_T(\partial F)$".
p. 81, Problem 6.1.4. \[ \begin{bmatrix} m \\ b \end{bmatrix} \rightarrow \begin{bmatrix} m \\ \bar{b} \end{bmatrix} \].

p. 82, first paragraph "substitute the result in \( x^2/8 + y^2/2 \) → "substitute the result in \( xy \)."

p. 82, Theorem 6.7, proof.
"Let \( x(\xi) \) parameterize . . . \( x(t) = x(\xi(t)) \) parameterize a curve " → "Let \( x(t) \) be a parameterized curve".

p. 83, Problem 6.2.3. "on the triangle" → "inside the triangle".

Ans. Max 20, Min 4.

p. 89, last line of the displayed equation in the proof of Theorem 7.3 \[ |P| \rightarrow 0 \] to \[ |P| \rightarrow 0 \].

p. 89. Replace the bulleted points with
- A definite integral of \( f \), a number. It is the limit of sums (Definition 7.1). The number can represent areas, masses, etc.
- An indefinite integral of \( f \), a function. It is an \( F \) such that \( F' = f \).

p. 95, Definition 7.8. "tangent line to \( S \)" → "tangent line to \( C \)".

p. 117, Problem 8.2.3. Should read \( x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1 \).

p. 119, first two paragraphs. \( f \rightarrow F \) in the integrands.

p. 122, Corollary 9.4, statement. "\( S \subset \mathbb{R}^n \) → \( S \subset \mathbb{R}^3 \)."

p. 130, Exercise 10.2.
"defined on the boundary" → "defined on \( M \) and \( \partial M \)."

p. 133, toward bottom. "which is wanting in the definition given by Eq. (5.4)."

p. 134, Problem 10.2.4b. Change vector field \( f \) to scalar field \( f \). Drop Part c.

p. 136, Corollary 10.5, Proof. "Step (2) uses LAGA Theorem 6.30c and \( dS^* = d\sigma \)." → "Step (2) uses LAGA Theorem 6.30c."

p. 138, toward bottom.
"which is wanting in the definition given by Eq. (5.5)."

p. 141, top. "manifolds of arbitrary dimension." → "\( \mathbb{R}^m \)."

p. 142, Theorem 10.10, statement. Serious error!
"Let \( M \) be an \( m \)-dimensional manifold." → "Let \( M \) be a bounded open set in \( \mathbb{R}^m \)."

\[ F(x_0) = \frac{(-1)^m+1}{\Omega_m} \int_{\partial M} \frac{x - x_0}{|x - x_0|^m} d^{m-1}x F(x) , \]

Corollary 10.11, statement.
"Let \( M \) be a manifold." → "Let \( M \) be a bounded open set in \( \mathbb{R}^m \)."

p. 148, Theorem 11.5, first sentence of statement. Change to "Let \( x(s) \) and \( \bar{x}(s) \), \( 0 \leq s \leq L \), parameterize curves \( C \) and \( \bar{C} \)."

p. 150, Theorem 11.8, proof. \( \ell(C) = \int_{[a,b]} |x'(u_1(t), u_2(t))| \, dt \)
\[ |x'(u_1, u_2)|^2 = x'(u_1, u_2) \cdot x'(u_1, u_2) = \]
p. 152, Corollary 11.12, proof. \( f'(h) \to f'(h) \).

p. 152, Exercise 11.32a. Show that the metric \( G(r, \theta) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \).

p. 161, Figure 11.7. Remove the hat on \( p_1 \) and \( p_2 \).

p. 163, Standard Terminology. "The metric \( G \) (Eq. (11.5))" \( \to \) "The expression \( ds^2 = g_{ij} du_i du_j \) (Eq. (11.7))".

p. 172, Differentiation entry. "print \( \text{diff(diff(x**2,x),y)} \)" \( \to \) "print \( \text{diff(diff(y*x**2,x),y)} \)".

p. 172, Jacobian entry. Redo:

\begin{itemize}
  \item Jacobian. Let \( X \) be an \( m \times 1 \) matrix of \( m \) variables. Let \( Y \) be an \( n \times 1 \) matrix of functions of the \( m \) variables. These define a function \( f : X \in \mathbb{R}^m \mapsto Y \in \mathbb{R}^n \). Then \( Y.jacobian(X) \) is the \( n \times m \) matrix of \( f'_x \), the differential of \( f \).
\end{itemize}

\( r, \theta = \text{symbols('r theta')} \)
\( X = \text{Matrix([r, theta])} \)
\( Y = \text{Matrix([r*cos(theta), r*sin(theta)])} \)
\( \text{print } Y.jacobian(X) \quad \# \text{ Print } 2 \times 2 \text{ Jacobian matrix.} \)
\( \text{print } Y.jacobian(X).\text{det()} \quad \# \text{ Print Jacobian determinant (only if } m = n). \)

Sometimes you want to differentiate \( Y \) only with respect to some of the variables in \( X \), for example when applying Eq. (3.24). Then include only those variables in \( X \). For example, using \( X = \text{Matrix([r])} \) in the example above produces the \( 2 \times 1 \) matrix \( [\cos \theta] \).

p. 172, Iterated Integrals entry.

"\text{make_symbols('x y')}" \( \to \) "\( x, y = \text{symbols('x y')} \)".

p. 174, Reciprocal Basis entry. Drop everything after the first sentence.

p. 175. Change to

Compute the vector derivative \( \partial f \), divergence \( \partial \cdot f \), curl \( \partial \wedge f \):
\( M.\text{grad } * f \), \( M.\text{grad } < f \), \( M.\text{grad } \wedge f \).