Let $p$ be a permutation of $[1 \ 2 \ \ldots \ \ n]$. Let $G(p)$ be the number of times in $p$ that a number is greater than a number to its right. For example, $G([2 \ 4 \ 1 \ 3]) = 3$. Note that $G([1 \ 2 \ \ldots \ \ n]) = 0$. A transposition of numbers in adjacent positions changes $G$ by $\pm 1$. Every transposition can be expressed as a product of an odd number of such transpositions. Therefore every transposition changes the parity of $G$. Thus the number of transpositions used to obtain $p$ is always even or always odd, according as $G(p)$ is even or odd.