Errata for *Vector and Geometric Calculus*
both printings

p. 30, Exercise 3.13. \( f: U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m \rightarrow f: \mathbb{R}^n \rightarrow \mathbb{R}^m \).

p. 38, Problem 3.4.12. Change to “The ideal gas law is \( pv = nRT \), where \( n, p, v, T \) are the number of moles, pressure . . . .”

p. 49, Definition 4.1.
“Let \( q \in A \) and set \( p = x(q) \).” → “Let \( t \in A \) and set \( p = x(t) \).”

p. 51, Exercise 4.9. Change the definition: \( \omega = -\Omega^* \).

p. 55, Figure 4.9, caption. “onto \( S \)” → “to \( S \)”

p. 59, Corollary 5.7. \( f^*(b) \rightarrow f_x^*(b) \).

p. 64, bottom. Problem 4.3.12 → LAGA Problem 4.3.12

p. 72, Exercise 5.23. Equations should read
\[
\dot{\phi} = \cos \phi (\cos \theta i + \sin \theta j) - \sin \phi k, \\
\dot{\theta} = -\sin \theta i + \cos \theta j.
\]

p. 75, footnote. \( \mathbb{R}^3 \rightarrow \mathbb{R}^n \).

p. 76, Problem 5.5.1b, second printing only. “even though \( \hat{h} \) is” → “even though \( \hat{t} \) is”.

p. 79, Theorem 6.5. A much better proof:
Proof. Since \( \nabla f(x) = 0 \), \( \partial_i f(x) h_i = 0 \). And \( \partial_i f(x) h_i h_j > 0 \) for \( h \neq 0 \), since \( Hf(x) \) is positive definite. Then \( \partial_i f(x + t^*h) h_i h_j > 0 \) for small \( t^*h \neq 0 \), since the partial derivatives are continuous at \( x \). The theorem now follows from Eq. (3.2). \( \square \)

p. 91, after *Think of*. “Divide \( C \) into infinitesimal parts. Multiply the value of \( F \) on each part by the infinitesimal length \( ds \) of the part. Add to form the integral.”

p. 101, Exercise 7.24b. “not conservative” → “not conservative in \( \mathbb{R}^2 - \{0\} \)”.

p. 102, following Definition 7.16. “All simply closed curves” → “All closed curves”.

p. 112, Exercise 8.2. \( \int_{y=0}^1 \rightarrow \int_{y=0}^2 \).

p. 116, Exercise 8.8. The answer is \( \pi e^4 \).

p. 119, after Definition 9.1. “infinitesimal area \( dS \)”.

p. 142, below Theorem 10.10. \( f(x) = \int_a^x f'(t)dt + f(a) \).

p. 148, Theorem 11.5, Proof. \( \ddot{x}(s) \rightarrow \ddot{x}(s) \), twice
Errata for *Vector and Geometric Calculus*  
first printing only

Due to publisher error the shading in several of figures is washed out in some copies of the book. The correct shading is shown below. I think that if Figure 2.1 is OK in your book, then all figures are.

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p. 4, Exercise 1.1. “c_{ij} \rightarrow c_{ik}”.

p. 6, Problem 1.1.1. “x(t) \rightarrow x(\theta)”.
p. 8, Exercise 1.11. Delete “We will do this often.” Add “This will allow us to specialize formulas for surfaces defined parametrically to surfaces defined by $z = f(x, y)$. Exercise 5.25 is an example.”

p. 11, Exercise 1.14. “$x \neq 0$” $\rightarrow$ “$x > 0$”.

\[
\phi = \arccos(z/r) \rightarrow \phi = \arctan(r/z).
\]

p. 23, Exercise 3.1. “RE$^{m}$ to $\mathbb{R}^{n}$” $\rightarrow$ “$\mathbb{R}^{m}$ to $\mathbb{R}^{n}$”.

p. 33, Problem 3.3.1. Append the sentence “Then for fixed $x$, the differential is the linear transformation $h \mapsto f'(x)h$.”

p. 33, Problem 3.3.3. $(\rho, \theta, \phi)$ $\rightarrow$ $(\rho, \phi, \theta)$

p. 34, first and second displayed equations should read
\[
x, h \in \mathbb{R}^{n} \Rightarrow (g \circ f)(x) \in \mathbb{R}^{p} \Rightarrow (g \circ f)'(x)(h) \in \mathbb{R}^{p},
x, h \in \mathbb{R}^{n} \Rightarrow f'(x)(h) \in \mathbb{R}^{m} \Rightarrow (g'(f(x)) \circ f'(x))(h) \in \mathbb{R}^{p}.
\]

p. 34. Line should read $\frac{\Delta}{\Delta} \ldots + [g'(R(h)) |h| + S(kh)] |kh|$.

p. 34. Replace the end of the page with the following:
The added phrase “divided by $|h|$” is the reason for the changes.
To finish, we show that the term in brackets above, divided by $|h|$, approaches zero with $|h|$. First, using the continuity of $g'$ (Theorem 2.10),
\[
\lim_{h \to 0} g'(R(h)) = g'(\lim_{h \to 0} R(h)) = g'(0) = 0.
\]

Second, with $|f'|_{C}$ the operator norm of $f'$,
\[
|kh| \leq |f'(h)| + |R(h)||h| \leq |f'|_{C}|h| + |R(h)||h|.
\]
Thus, since $(h \to 0) \Rightarrow (kh \to 0) \Rightarrow (S(kh) \to 0)$, $\lim_{h \to 0} |S(kh)||kh|/|h| = 0$.

p. 37, statement of Theorem 3.12.
“Then the inverse function $(f^{-1})^{-1}$” $\rightarrow$ “Then the inverse function $f^{-1}$”.

p. 40, following Theorem 3.15: “In other words, $\partial_{h}f$ is linear in both $h$ and $f$.”

p. 40, Problem 3.5.5. The variable names I used lead to confusion. Change to $f(x, y) = (x \cos y, x \sin y)$. And add “(All coordinates are cartesian.)”

p. 40, Problem 3.5.3. “continuity of $f$ at $x$.” $\rightarrow$ “continuity of $f$ at $x$.”

p. 41, Statement of Theorem 3.17.
“has a differentiable inverse” $\rightarrow$ “has a continuously differentiable inverse”.

p. 42, third line. Remove “there is a neighborhood of each $y_{i}$ in which”.

p. 45, Exercise 3.6.1b. “Determine $\partial \rho/\partial z$.”

p. 48, below Eq. (4.3). “of higher dimension” $\rightarrow$ “in higher dimensions”.

pp. 49-55. Equation (4.4) in Section 4.1 established the notation of $m$-dimensional manifolds $M$ as subsets of $\mathbb{R}^{n}$. However, Sections 4.2 and 4.3, while mostly internally consistent, are inconsistent with this notation. The following changes remove the inconsistency:
p. 49, Paragraph 2. "Let \( x(t) \) parameterize a curve \( C \).
Then \( x \) has a nonzero differential (p. 48, bottom). By Problem 3.3.1 it is . . . ”

p. 50, line after Theorem 4.3. “Definition 4.5” → “Eq. (4.5)”

p. 52, Problem 4.2.4.
   a. Show that components of \( \Omega \) must have grade 2 or 3.
   b. Show that components of \( \Omega \) must have grade 0 or 2.

p. 53, Eq. (4.10). “\( \lim_{h \to 0} \)” → “\( \lim_{h \to 0} \)”

p. 53, sentence below Definition 4.6. “Recall that the differential \( x'_q \) is one-to-one
and maps linearly independent vectors ...”

p. 55, Theorem 4.7. “\( f'_p \) is one-to-one.” → “\( f'_p \) restricted to \( T_p \) is one-to-one.”

p. 56, Problem 5.2.5. Change to \( \nabla \cdot \beta = \frac{1}{\rho} \nabla f(\rho) + |\rho| f'(\rho) / \rho \).

p. 57, Exercise 5.14a. \( \nabla \cdot f = \partial_1 f_1 + \partial_2 f_2 \).

p. 71, Paragraph 4. In general: (i). Each basis vector \( x^k \) is orthogonal to the
surface formed by fixing the coordinate \( u^k \). (ii). Each basis vector \( x_j \) is tangent to
the curve which is the intersection of the two surfaces formed by fixing in turn the
coordinates other than \( u^j \).

p. 72, formula for \( \nabla f \) in cylindrical coordinates. “\( r^{-1} \partial f/\partial \phi \) → “\( r^{-1} \partial f/\partial \phi \)”

p. 74, Problem 5.4.2. Better: \( \text{Hint: If } B \text{ is a blade, then } B^{-1} = B/B^2, \text{ where } B^2 \text{ is a scalar.} \)

p. 74, Problem 5.4.5. Delete. Renumber Problems 5.4.6-5.4.8 to 5.4.5-5.4.7.

p. 75, line 3. “manifolds in \( \mathbb{R}^m \) → “manifolds in \( \mathbb{R}^n \)”

p. 76, Problem 5.5.1. “to the unit sphere” → “in \( \mathbb{R}^2 \)”
“\( D = P_T(\partial) \)” → “\( DF = P_T(\partial F) \)”

p. 76, Problem 5.5.2. New Part (b): Define a directional derivative for fields
defined on a surface by \( \partial_h f(p) = (h \cdot \partial) f(p) \) (Definition 11.14). Compute \( \partial_t t \) on the equator. \( \text{Ans. } -(\sin \theta i + \cos \theta j) / \rho. \)

Note that at the equator \( t \) is in the tangent plane but \( \partial_t t \) is not.
p. 81, Eq. (??). $\begin{bmatrix} m \\ b \end{bmatrix} \rightarrow \begin{bmatrix} \bar{m} \\ \bar{b} \end{bmatrix}$.

p. 82, first paragraph “substitute the result in $x^2/8 + y^2/2$” → “substitute the result in $xy$”

p. 82, Theorem 6.7, proof.

“Let $x(\xi)$ parameterize . . . $x(t) = x(\xi(t))$ parameterize a curve” → “Let $x(t)$ be a parameterized curve”.

p. 83, Problem 6.2.3. “on the triangle” → “inside the triangle”.

Ans. Max 20, Min 4.

p. 89, last line of the displayed equation in the proof of Theorem 7.3 $|P| \rightarrow 0$ to $|P| \rightarrow 0$.

p. 89. Replace the bulleted points with

- A definite integral of $f$, a number. It is the limit of sums (Definition 7.1). The number can represent areas, masses, etc.
- An indefinite integral of $f$, a function. It is an $F$ such that $F' = f$.

p. 95, Definition 7.8. “tangent line to $S$” → “tangent line to $C$”.

p. 117, Problem 8.2.3. Should read $x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1$.

p. 119, first two paragraphs. $f \rightarrow F$ in the integrands.

p. 122, Corollary 9.4, statement. “$S \subset \mathbb{R}^n$” → “$S \subset \mathbb{R}^3$.”

p. 130, Exercise 10.2.

“defined on the boundary” → “defined on $M$ and $\partial M$”.

p. 133, toward bottom. “which is wanting in the definition given by Eq. (5.4).”

p. 134, Problem 10.2.4b. Change vector field $f$ to scalar field $f$. Drop Part c.

p. 136, Corollary 10.5, Proof. “Step (2) uses LAGA Theorem 6.23c and $dS^* = d\sigma$.,”

→ “Step (2) uses LAGA Theorem 6.23c.”

p. 138, toward bottom.

“which is wanting in the definition given by Eq. (5.5).”

p. 141, top. “manifolds of arbitrary dimension.” → “$\mathbb{R}^m$.”

p. 142, Theorem 10.10, statement. Serious error!

“Let $M$ be an $m$-dimensional manifold.” → “Let $M$ be a bounded open set in $\mathbb{R}^m$.”

$p. 150, \text{Theorem } 11.8, \text{proof. } \ell(C) = \int_{[a,b]} |x'(u_1(t), u_2(t))| \, dt$ 

$|x'(u_1, u_2)|^2 = x'(u_1, u_2) \cdot x'(u_1, u_2)$

$p. 152, \text{Corollary } 11.12, \text{proof. } f'(h) \rightarrow f_\nu'(h)$.

p. 152, Exercise 11.32a. Show that the metric $G(r, \theta) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
p. 161, Figure 11.7. Remove the hat on $p_1$ and $p_2$.

p. 163, Standard Terminology. “The metric $G$ (Eq. (11.5))” → “The expression $ds^2 = g_{ij} du_i du_j$ (Eq. (11.7)).”

p. 172, Differentiation entry. “print diff(diff(x**2,x),y)” → “print diff(diff(y*x**2,x),y)”.

p. 172, Jacobian entry. Redo:

Jacobian. Let $X$ be an $m \times 1$ matrix of $m$ variables. Let $Y$ be an $n \times 1$ matrix of functions of the $m$ variables. These define a function $f: X \in \mathbb{R}^m \mapsto Y \in \mathbb{R}^n$. Then $Y.\text{jacobian}(X)$ is the $n \times m$ matrix of $f_x'$, the differential of $f$.

\[
r, \theta = \text{symbols}('r \theta')
\]
\[
X = \text{Matrix}([r, \theta])
\]
\[
Y = \text{Matrix}([r*\cos(\theta), r*\sin(\theta)])
\]
\[
\text{print} \ Y.\text{jacobian}(X) \quad \# \text{Print 2} \times 2 \text{ Jacobian matrix.}
\]
\[
\text{print} \ Y.\text{jacobian}(X).\text{det()} \quad \# \text{Print Jacobian determinant (only if } m = n).\]

Sometimes you want to differentiate $Y$ only with respect to some of the variables in $X$, for example when applying Eq. (3.25). Then include only those variables in $X$. For example, using $X = \text{Matrix}([r])$ in the example above produces the $2 \times 1$ matrix $\begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$.

p. 172, Iterated Integrals entry.

“make_symbols('x y')” → “x, y = symbols('x y').”

p. 174, Reciprocal Basis entry. Drop everything after the first sentence.

p. 175. Change to

Compute the vector derivative $\partial f$, divergence $\partial \cdot f$, curl $\partial \wedge f$:

$M.\text{grad} \ast f$, $M.\text{grad} < f$, $M.\text{grad} \wedge f$. 