Assume:
(A) The speed of light is the same in all inertial frames. (Take \( c = 1 \).)
(B) A clock moving with constant velocity \( v \) in an inertial frame \( I \) runs at a constant rate \( \gamma = \gamma(|v|) \) with respect to the synchronized clocks of \( I \) which it passes.

Assumption (B) follows directly from the relativity principle. We do not assume that the Lorentz transformation is linear.

In the figure, \( E \) is an arbitrary event plotted in an inertial frame \( I \), \( L^+ \) and \( L^- \) are the two light worldlines through \( E \), and \( O \) is the worldline of the spatial origin of an inertial frame \( I' \) moving with velocity \( v \) in \( I \). On \( O \),

\[
X' = 0, \quad X = vT, \quad \text{and} \quad T = \gamma T'.
\]

Thus on \( O \),

\[
\begin{align*}
T + X &= \gamma(1 + v)(T' + X') \quad (1) \\
T - X &= \gamma(1 - v)(T' - X'). \quad (2)
\end{align*}
\]

Since \( c = 1 \) in \( I \), an increase in \( T \) along \( L^- \) is accompanied by an equal decrease in \( X \). Thus \( T + X \) is the same at \( E \) and \( F \). Likewise, since \( c = 1 \) in \( I' \), \( T' + X' \) is the same at \( E \) and \( F \). Thus Eq. (1), which is true at \( F \), is also true at \( E \). Similar reasoning using \( L^+ \) proves Eq. (2) true at \( E \). Add and subtract Eqs. (1) and (2):

\[
\begin{align*}
T &= \gamma(T' + vX') \quad (3) \\
X &= \gamma(vT' + X'). \quad (4)
\end{align*}
\]

For \( X = 0 \) in Eq. (4), \( X' = -vT' \); the origin of \( I \) has velocity \(-v \) in \( I' \). Thus, switching \( I \) and \( I' \) and using (B), the reasoning for Eq. (1) also gives \( T' + X' = \gamma(1 - v)(T + X) \). Substituting this in Eq. (1) gives \( \gamma = (1 - v^2)^{-\frac{1}{2}} \).

---