

Atmospheric Moisture Equations

Symbols List:

L_v	=	$2.50 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$	for water (vaporization)
L_d	=	$2.83 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$	for ice (sublimation)
\mathcal{R}_v	=	$461 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$	(gas constant for water vapor)
\mathcal{R}_d	=	$287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$	(gas constant for dry air)
ϵ	=	$\mathcal{R}_d / \mathcal{R}_v = 0.622$	$\text{g}_{\text{vapor}} / \text{g}_{\text{dry air}}$
ρ_d	=	density of dry air at sea level and 20° C	$= 1.22 \text{ kg} \cdot \text{m}^{-3}$
P	=	total atmospheric pressure	
T	=	parcel temperature	
e	=	vapor partial pressure	
T_0	=	273 K	
e_0	=	$0.611 \text{ kPa} = 6.11 \text{ mb}$	or hPa

◇ **Saturation Vapor Pressure e_s**

Clausius-Clapeyron equation for atmospheric conditions

$$\frac{de_s}{dT} = \frac{L_v(T)e_s}{R_v T^2}$$

Solving the differential equation results in the following expression for saturation vapor pressure e_s as a function of temperature T :

$$e_s = e_0 \exp \left[\frac{L}{\mathcal{R}_v} \left(\frac{T - T_0}{T_0 \cdot T} \right) \right] \quad \text{units of } e_0, \quad L = L_v \text{ for water surfaces, and } L_d \text{ for ice surfaces}$$

◇ **Mixing Ratio MR**

$$MR = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \frac{\epsilon \cdot e}{P - e} \quad \text{units} \quad \frac{\text{g of vapor}}{\text{g of dry air}}$$

◇ **Specific Humidity SH**

$$SH = \frac{\text{mass of water vapor}}{\text{total mass of air}} = \frac{\epsilon \cdot e}{P} \quad \text{units} \quad \frac{\text{g of vapor}}{\text{g of air}}$$

◇ **Absolute Humidity AH**

$$AH = \frac{\text{mass of water vapor}}{\text{volume of air}} = \frac{e}{\mathcal{R}_v T} = \frac{e}{P} \epsilon \rho_d \quad \text{units} \quad \frac{\text{kg of vapor}}{\text{m}^3}$$

◇ **Relative Humidity RH**

$$RH = \frac{e}{e_s} \times 100\% = \frac{MR}{MR_s} \times 100\% = \frac{SH}{SH_s} \times 100\% = \frac{AH}{AH_s} \times 100\%$$

$()_s$ = saturation value for the given quantity

◇ **Virtual Temperature T_v**

$$T_v = T(1 + 0.61MR^*)$$

* - the mixing ratio (MR or W) must be in kg/kg, not g/kg

SVP and SMR versus Temperature

Temperature		SVP (e_s) mb	Saturation Mixing Ratio (g/kg)			
$^{\circ}$ C	$^{\circ}$ F		1000 mb	850 mb	700 mb	500 mb
-20	-4.0	1.3	0.8	0.9	1.1	1.6
-19	-2.2	1.4	0.9	1.0	1.2	1.7
-18	-0.4	1.5	0.9	1.1	1.3	1.9
-17	1.4	1.6	1.0	1.2	1.5	2.0
-16	3.2	1.8	1.1	1.3	1.6	2.2
-15	5.0	1.9	1.2	1.4	1.7	2.4
-14	6.8	2.1	1.3	1.5	1.9	2.6
-13	8.6	2.3	1.4	1.7	2.0	2.8
-12	10.4	2.5	1.5	1.8	2.2	3.1
-11	12.2	2.7	1.7	1.9	2.4	3.3
-10	14.0	2.9	1.8	2.1	2.6	3.6
-9	15.8	3.1	1.9	2.3	2.8	3.9
-8	17.6	3.4	2.1	2.5	3.0	4.2
-7	19.4	3.6	2.3	2.7	3.2	4.5
-6	21.2	3.9	2.4	2.9	3.5	4.9
-5	23.0	4.2	2.6	3.1	3.8	5.3
-4	24.8	4.5	2.8	3.3	4.1	5.7
-3	26.6	4.9	3.1	3.6	4.4	6.2
-2	28.4	5.3	3.3	3.9	4.7	6.6
-1	30.2	5.7	3.6	4.2	5.1	7.1
0	32.0	6.1	3.8	4.5	5.5	7.7
1	33.8	6.6	4.1	4.8	5.9	8.3
2	35.6	7.1	4.4	5.2	6.3	8.9
3	37.4	7.6	4.8	5.6	6.8	9.6
4	39.2	8.1	5.1	6.0	7.3	10.3
5	41.0	8.7	5.5	6.5	7.9	11.1
6	42.8	9.4	5.9	6.9	8.4	11.9
7	44.6	10.0	6.3	7.4	9.1	12.7
8	46.4	10.8	6.8	8.0	9.7	13.7
9	48.2	11.5	7.2	8.5	10.4	14.7
10	50.0	12.3	7.8	9.2	11.2	15.7
11	51.8	13.2	8.3	9.8	11.9	16.9
12	53.6	14.1	8.9	10.5	12.8	18.1
13	55.4	15.1	9.5	11.2	13.7	19.3
14	57.2	16.1	10.2	12.0	14.6	20.7
15	59.0	17.2	10.9	12.8	15.7	22.2
16	60.8	18.4	11.6	13.7	16.7	23.7
17	62.6	19.6	12.4	14.7	17.9	25.3
18	64.4	20.9	13.3	15.7	19.1	27.1
19	66.2	22.3	14.2	16.7	20.4	29.0
20	68.0	23.7	15.1	17.8	21.8	31.0
21	69.8	25.2	16.1	19.0	23.3	33.1
22	71.6	26.9	17.2	20.3	24.8	35.3
23	73.4	28.6	18.3	21.7	26.5	37.7
24	75.2	30.4	19.5	23.1	28.3	40.3
25	77.0	32.3	20.8	24.6	30.1	43.0
26	78.8	34.4	22.1	26.2	32.1	45.9
27	80.6	36.5	23.6	27.9	34.2	49.0
28	82.4	38.8	25.1	29.7	36.5	52.3
29	84.2	41.2	26.7	31.7	38.9	55.8
30	86.0	43.7	28.4	33.7	41.4	59.5
31	87.8	46.3	30.2	35.8	44.1	63.5
32	89.6	49.1	32.1	38.1	46.9	67.7
33	91.4	52.0	34.2	40.6	50.0	72.3
34	93.2	55.1	36.3	43.1	53.2	77.1
35	95.0	58.4	38.6	45.9	56.6	82.3
36	96.8	61.8	41.0	48.8	60.3	87.8
37	98.6	65.4	43.5	51.9	64.1	93.6
38	100.4	69.2	46.2	55.1	68.2	99.9
39	102.2	73.2	49.1	58.6	72.6	106.7
40	104.0	77.4	52.2	62.3	77.3	113.9