# Atmospheric Moisture Equations

**Symbols List**:

$L_v$ = $2.50\times10^6$ J· $kg^{-1}$ for water (vaporization)
$L_d$ = $2.83\times10^6$ J· $kg^{-1}$ for ice (sublimation)
$\mathcal{R}_v$ = 461 $J{\cdot}K^{-1}\cdot kg^{-1}$ (gas constant for water vapor)
$\mathcal{R}_d$ = 287 $J{\cdot}K^{-1}\cdot kg^{-1}$ (gas constant for dry air)
$\epsilon$ = $\mathcal{R}_d/\mathcal{R}_v = 0.622$ $g_{vapor}/g_{dry\ air}$
$\rho_d$ = density of dry air at sea level and 20 ° C = 1.22 kg· $m^{-3}$
$P$ = total atmospheric pressure
$T$ = parcel temperature
$e$ = vapor partial pressure
$T_0$ = 273 K
$e_0$ = 0.611 kPa = 6.11 mb or hPa

◇ **Saturation Vapor Pressure** $e_s$

Clausius-Clapeyron equation for atmospheric conditions

$$\frac{de_s}{dT} = \frac{L_v(T)e_s}{R_vT^2}$$

Solving the differential equation results in the following expression for saturation vapor pressure $e_s$ as a function of temperature $T$:

$$e_s = e_0\exp\left[\frac{L}{\mathcal{R}_v}\left(\frac{T-T_0}{T_0\cdot T}\right)\right] \overset{\text{units}}{=} \text{units of } e_0, \quad L = L_v \text{ for water surfaces, and } L_d \text{ for ice surfaces}$$

◇ **Mixing Ratio** $MR$

$$MR = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \frac{\epsilon\cdot e}{P-e} \overset{\text{units}}{=} \frac{\text{g of vapor}}{\text{g of dry air}}$$

◇ **Specific Humidity** $SH$

$$SH = \frac{\text{mass of water vapor}}{\text{total mass of air}} = \frac{\epsilon\cdot e}{P} \overset{\text{units}}{=} \frac{\text{g of vapor}}{\text{g of air}}$$

◇ **Absolute Humidity** $AH$

$$AH = \frac{\text{mass of water vapor}}{\text{volume of air}} = \frac{e}{\mathcal{R}_vT} = \frac{e}{P}\ \epsilon\rho_d \overset{\text{units}}{=} \frac{\text{kg of vapor}}{\text{m}^3}$$

◇ **Relative Humidity** $RH$

$$RH = \frac{e}{e_s}\times100\% = \frac{MR}{MR_s}\times100\% = \frac{SH}{SH_s}\times100\% = \frac{AH}{AH_s}\times100\%$$

$()_s$ = saturation value for the given quantity

◇ **Virtual Temperature** $T_v$

$$T_v = T(1+0.61MR^*)$$

* - the mixing ratio ($MR$ or $W$)must be in kg/kg, not g/kg